

A sliding mode control for a wound rotor synchronous generator with an isolated RL load

R.S. Muñoz-Aguilar, A. Dòria-Cerezo and E. Fossas

Abstract—In this paper a sliding mode controller for the stator voltage amplitude of a stand-alone wound rotor synchronous generator is presented. The standard dq-model of the electrical machine connected to an isolated inductive load is obtained. Then, a control law is designed in the framework of sliding mode control. The controller introduces a dynamic extension so that the stator amplitude relative degree is one. Numerical simulations are also presented to validate the presented control law.

I. INTRODUCTION

The wound rotor synchronous machine (WRS) is a doubly-fed electrical machine which can be used for both: generation [2][3] and driving applications [1][8]. Usually, wound rotor synchronous generators are studied when they are directly connected to the power grid [2]. In this case, the stator voltage and frequency are set by the power grid, while the rotor voltage helps to improve the power factor and to compensate the reactive power at the connection point. In this paper we study the isolated load case which is significantly different: neither amplitude nor frequency of stator voltage are fixed. For the stand-alone configuration, although the mechanical speed determines the frequency, the rotor voltage is used to set the stator amplitude.

The WRS is controlled by several techniques, in industry the most common are linear techniques [5][9], however, decoupling methods [4], which widely employed for asynchronous machines, are also extended in the synchronous case. Modern non-linear techniques such as passivity-based control [1] or predictive control [7], are also used for regulating this kind of machine.

Sliding Mode Control (SMC) has been proposed as an appropriate technique for controlling electrical machines [10]. For instance, the classical approach of the SMC was used for a position servo system [6]. More recently, sliding observers combined with control linearization and singular perturbations, are applied to a synchronous generator connected to a power grid [2]. Higher order sliding modes, have also been proposed in [11] for avoiding chattering in a wind energy generator module power control.

The main goal of this work is to design a sliding mode control algorithm for a wound rotor synchronous generator feeding an isolated resistive-inductive unknown load.

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In this case, sliding mode control cannot be directly applied to the system natural output because it is relative degree zero. Instead, a dynamical extension of the system is proposed so that the output relative degree is one.

The paper is organized as follows. In Section II the wound rotor synchronous machine model is presented and the control goals are described. Section III concerns the Sliding Mode controller design, while simulation results are shown in Section IV. Finally, conclusions are drawn in Section V.

II. SYSTEM DESCRIPTION

In this Section the dynamical model of the system is presented. Figure 1 shows the following scenario: an Internal Combustion Engine (ICE) drags a WRS, which acts as a generator to feed an isolated load.

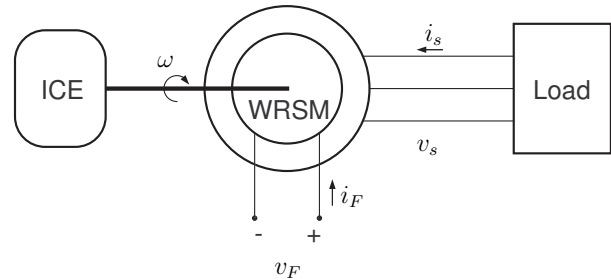


Fig. 1. Scheme of an isolated wound rotor synchronous machine with an isolated load.

As explained before, this system differs from the typical grid connection, in which, the frequency and the voltage amplitude are set by the grid. In an isolated connection the frequency is determined by the mechanical speed (provided by the ICE), while the voltage amplitude must be assured by the rotor field voltage.

A. Dynamical model

From the well-known WRS dynamical equations (in the dq coordinates), and the interconnection rules with an inductive load, the whole dynamic system is presented.

Assuming that the mechanical speed is externally regulated by the ICE, the electrical part of the wound rotor synchronous machine can be described as

$$L \frac{dx}{dt} = \begin{pmatrix} -R_s & \omega L_s & 0 \\ -\omega L_s & -R_s & -\omega L_m \\ 0 & 0 & -R_F \end{pmatrix} x + \begin{pmatrix} v_d \\ v_q \\ v_F \end{pmatrix} \quad (1)$$

where

$$L = \begin{pmatrix} L_s & 0 & L_m \\ 0 & L_s & 0 \\ L_m & 0 & L_F \end{pmatrix}$$

is the inductance matrix, $x^T = (i_d, i_q, i_F) \in \mathbb{R}^3$ are the dq-stator and field currents, R_s and R_F are the stator and field resistances, L_s , L_m and L_F are the stator, mutual and field inductances, ω is the mechanical speed¹, v_d , v_q and v_F are the dq-stator voltages and the field voltage which will be used as a control input.

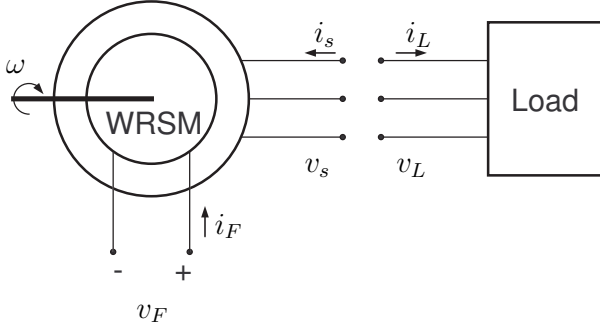


Fig. 2. Detail of the interconnection between a wound rotor synchronous machine and an inductive load.

In order to design the control law, let us obtain the complete model of a WRS connected to an inductive load, which is modeled as a pure resistive element, R_L , in series with a pure inductive element, L_L . The interconnection scheme is depicted in Figure 2, where $v_L^T = (v_{Ld}, v_{Lq}) \in \mathbb{R}^2$ and $i_L^T = (i_{Ld}, i_{Lq}) \in \mathbb{R}^2$ are the dq load voltages and currents, which are related by

$$v_L = (R_L I_2 + \omega L_L J_2) i_L + L_L \frac{d}{dt} i_L, \quad (2)$$

where

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad J_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The interconnection rules, according to Fig. 2, are

$$\begin{aligned} v_s &= v_L \\ i_L &= -i_s. \end{aligned} \quad (3)$$

Now, putting together (1), (2) and (3), the system can be written in an affine form as

$$\hat{L} \frac{dx}{dt} = Ax + Gv_F, \quad (4)$$

where \hat{L} is a new inductance matrix,

$$\hat{L} = \begin{pmatrix} L_s + L_L & 0 & L_m \\ 0 & L_s + L_L & 0 \\ L_m & 0 & L_F \end{pmatrix}$$

¹For simplicity a two-poles machine is considered.

and the homogeneous dynamics A and the input vector G are given by

$$A = \begin{pmatrix} -(R_s + R_L) & \omega(L_s + L_L) & 0 \\ -\omega(L_s + L_L) & -(R_s + R_L) & -\omega L_m \\ 0 & 0 & -R_F \end{pmatrix},$$

and

$$G = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

B. Control objective

As mentioned before, this must ensure stator voltage amplitude and frequency. The synchronous machine stator frequency is directly given by the mechanical speed, which, in this paper is assumed to be constant and externally regulated. Then, the system output is the stator voltage amplitude V_s , which can be obtained, in a dq-framework as

$$V_s = \sqrt{v_d^2 + v_q^2}, \quad (5)$$

where v_d and v_q are defined by (2) and (3), and the current derivative in (2) can be obtained from (4). This leads to

$$\begin{aligned} \mu_d \frac{di_d}{dt} &= L_F(-(R_s + R_L)i_d + \omega(L_s + L_L)i_q) \\ &\quad - L_m(-R_F i_F + v_F) \end{aligned} \quad (6)$$

$$\begin{aligned} \mu_q \frac{di_q}{dt} &= -\omega(L_s + L_L)i_d - (R_s + R_L)i_q \\ &\quad - \omega L_m i_F. \end{aligned} \quad (7)$$

where $\mu_d = L_F(L_s + L_L) - L_m^2$ and $\mu_q = L_s + L_L$. Notice that, while the system dynamics is linear, as equation (4) states, the desired output, V_s , is a highly tedious nonlinear function, which can be obtained replacing (6) and (7) in (2), and then in (5). As explained before, the control input is the field voltage v_F .

Notice that for a pure resistive load the stator voltage is relative degree one in relation to v_F [3]. However, for isolated loads, including inductive terms, the stator voltage is relative degree zero. It is appropriate to mention here that to have relative degree one is a necessary condition for a switching surface to have sliding modes on it. Hence, the standard procedure used in the resistive case has to be modified.

III. CONTROL DESIGN

This section deals with the design of a sliding mode controller for an isolated wound rotor synchronous machine feeding an inductive load. As mentioned before, a system dynamics extension is carried out in order to have a relative degree one sliding surface. The ideal sliding dynamics is analyzed by means of the equivalent control.

A. Sliding surface and equivalent control

According to the control goals we define sliding surface $s(x)$ as

$$s(x) = V_s^2 - V_{ref}^2, \quad (8)$$

where V_{ref} is the stator voltage amplitude reference. Using (2), (5) and tacking into account (3), sliding surface can be given as a function of the state variables

$$s(x) = \left(-R_L i_d + \omega L_L i_q - L_L \frac{di_d}{dt} \right)^2 + \left(-R_L i_q - \omega L_L i_d - L_L \frac{di_q}{dt} \right)^2 - V_{ref}^2, \quad (9)$$

where the expressions for $\frac{di_d}{dt}$ and $\frac{di_q}{dt}$ are respectively given in (6) and (7).

As $\frac{di_d}{dt}$ depends on v_F , $s(x)$ is relative degree zero. Thus, a system dynamic extension is defined through equation (4) and

$$\frac{dv_F}{dt} = ku, \quad (10)$$

where k is a positive constant and the new state and input variables respectively are $z^T = (i_d, i_q, i_F, v_F)$ and u .

Let us to rewrite the extended system, as

$$\mathcal{L} \frac{dz}{dt} = \mathcal{A}z + \mathcal{G}u, \quad (11)$$

where

$$\mathcal{A} = \begin{pmatrix} -(R_s + R_L) & \omega(L_s + L_L) & 0 & 0 \\ -\omega(L_s + L_L) & -(R_s + R_L) & -\omega L_m & 0 \\ 0 & 0 & -R_F & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{L} = \begin{pmatrix} \hat{L} & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\mathcal{G} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k \end{pmatrix}.$$

The equivalent control, u_{eq} , is defined so that $\dot{s} = 0$,

$$\frac{ds}{dt} = \frac{\partial s}{\partial z} \frac{dz}{dt} = 0.$$

Using (11) this is equivalent to ,

$$\frac{\partial s}{\partial z} \mathcal{L}^{-1} (\mathcal{A}z + \mathcal{G}u_{eq}) = 0.$$

Hence

$$u_{eq} = - \left(\frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} \right)^{-1} \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{A}z. \quad (12)$$

From (8),

$$\frac{\partial s}{\partial z} = \frac{\partial v_d^2}{\partial z} + \frac{\partial v_q^2}{\partial z} = 2 \left(v_d \frac{\partial v_d}{\partial z} + v_q \frac{\partial v_q}{\partial z} \right)$$

Since \mathcal{G} has zeros in the first, second and third row, \mathcal{L}^{-1} contains only a nonzero term in the fourth row and v_q does not depends on v_F , we obtain

$$\begin{aligned} \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} &= v_d \frac{\partial}{\partial v_F} \left(L_L \frac{di_d}{dt} \right) k \\ &= 2k \frac{L_L L_m}{\mu_d} v_d, \end{aligned} \quad (13)$$

Hence,

$$u_{eq} = - \frac{\mu_d}{2k L_L L_m} \frac{1}{v_d} \left(\frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{A}z \right). \quad (14)$$

Note that the sliding motion can only be expected in the subspace defined by $v_d \neq 0$. This result was already obtained for the resistive case [3], where $v_d = -R_L i_d$, and

$$u_{eq} = - \frac{(L_s L_F - L_m^2)}{2 L_m R_L^2} \frac{1}{i_d} \left(\frac{\partial s}{\partial x} L^{-1} A x \right).$$

B. Equilibrium points

Equilibrium points of (11) are also a solution of (4)

$$x^*(v_F) = A^{-1} G v_F$$

which can be computed as

$$x^*(v_F) = \begin{bmatrix} -\frac{\omega^2 (L_s + L_L) L_m}{R_F ((R_s + R_L)^2 + \omega^2 (L_s + L_L)^2)} \\ -\frac{\omega (R_s + R_L) L_m}{R_F ((R_s + R_L)^2 + \omega^2 (L_s + L_L)^2)} \\ \frac{1}{R_F} \end{bmatrix} v_F. \quad (15)$$

To obtain v_F^* , (15) can be replaced into (9), yielding

$$v_F^* = \pm \frac{\sqrt{D} R_F V_{ref}}{(\omega^2 L_L^2 + R_L^2) \omega L_m} \quad (16)$$

where $D = (\omega^2 L_L^2 + R_L^2)((R_s + R_L)^2 + \omega^2 (L_s + L_L)^2)$. Finally (16) can be used in (15) to obtain i_d^* , i_q^* and i_F^* .

C. Ideal Sliding Dynamics

The Ideal Sliding Dynamics (ISD) is defined on the sliding surface presumed that it is invariant by the flow. From (11), replacing $u = u_{eq}$ together with $s(x) = 0$, the ISD could be rewritten as

$$\dot{x} = \hat{L}^{-1} (A x + G v_F(x)), \quad (17)$$

where now $v_F(x)$ comes from solving (9) for v_F .

Note also that the extended system is symmetric with respect to the origin. It is clear for the linear part of the dynamics defined by matrix A , while for the nonlinear part of $v_F(x)$ notice that $s(x, v_F) = s(-x, -v_F)$. Consequently, it is sufficient to analyze stability in one of the two equilibria given by (15) and (16).

As a first approximation, since this system is highly non-linear, the stability of the small-signal model around the equilibrium point will be analysed. Linearizing the ISD, (17), around the equilibrium point, we obtain

$$\begin{aligned} \hat{L} \dot{x} &\simeq \left(\frac{\partial}{\partial x} (A x + G v_F(x)) \Big|_{x=x^*} \right) (x - x^*) \\ &\quad + A x^* + G v_F(x^*). \end{aligned}$$

In this case the stability is given by the eigenvalues of

$$\hat{L}^{-1} \frac{\partial}{\partial x} (A x + G v_F(x)) \Big|_{x=x^*}$$

or

$$\hat{L}^{-1} \left(A + G \frac{\partial v_F}{\partial x} \Big|_{x=x^*} \right), \quad (18)$$

that have to lie in the left half plane. This analysis involves a tedious equation, but as a first approximation the stability

can be studied numerically. In simulation results, in Section IV, numerical computations have been done to determine the eigenvalues of (18).

D. Sliding Mode Controller

Sliding mode controller is in charge of approaching trajectories to the sliding surface and, when it is reached, the controller forces trajectories to remain on the surface (sliding modes). In order to define the control action for the extended system (11) and the sliding surface given in (8), let us consider the Lyapunov function candidate,

$$V = \frac{1}{2}s^2. \quad (19)$$

The discontinuous control action ought to guarantee $s\dot{s} \leq 0$, which is equivalent to,

$$s \frac{\partial s}{\partial z} \mathcal{L}^{-1} (Az + Gu) < 0.$$

Adding and subtracting $\mathcal{G}u_{eq}$ and taking into account (12), this equation is equivalent to

$$s \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} (u - u_{eq}) < 0. \quad (20)$$

Hence, the control action

$$u = u_{eq} - \text{sign} \left(s \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} \right),$$

yields

$$\dot{V} = - \left| s \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} \right| \leq 0.$$

If the control input u takes values in the discrete set $\{u_1, u_2\}$, where $u_1 < u_2$, the control law can be defined as

$$u = \begin{cases} u_1 & \text{if } s \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} > 0 \\ u_2 & \text{if } s \frac{\partial s}{\partial z} \mathcal{L}^{-1} \mathcal{G} \leq 0 \end{cases},$$

which also assures that $\dot{V} \leq 0$ in the in the sliding surface subset defined by $\min\{u_1, u_2\} \leq u_{eq} \leq \max\{u_1, u_2\}$. Using (13), and taking into account that $\frac{2kL_L L_m}{\mu_d} > 0$, the switching policy

$$u = \begin{cases} u_1 & \text{if } -sv_d > 0 \\ u_2 & \text{if } -sv_d \leq 0 \end{cases}$$

ensures $s\dot{s} \leq 0$ presumed that the equivalent control fulfills the later inequalities. Note that the (fictitious) control law, u , is a simple function which does not depend on the load values, neither on the machine parameters. Only v_d and v_q (which are measurable outputs) are required.

A more complicated task is to determine, analytically, the range of $[u_1, u_2]$ to ensure sliding. This is not trivial because u_{eq} is a complex function and it depends on the state values, see equation (14). A simple analysis shows that in the equilibrium $u_{eq}^* = 0$. Actually, effectively, from (14),

$$u_{eq}^* = -\frac{\mu_d}{2kL_L L_m} \frac{1}{v_d^*} \left(\frac{\partial s}{\partial z} \mathcal{L}^{-1} Az^* \right),$$

thus, replacing it on (11) yields $Az^* = 0$. Hence, the only condition to ensure sliding close to equilibria of the ideal sliding dynamics is $u_1 < 0 < u_2$.

The rotor voltage can be obtained from (10)

$$v_F = k \int u \, dt.$$

In a real application the actual control input, v_F has to lie in a given bounded set. Note that this should not necessarily constrain the values the fictitious variables u_1 and u_2 take.

Figure 3 shows the proposed control scheme. Note that, as a result of the control design, the error of the square of the stator voltage amplitude is multiplied by the v_d voltage. The resulting control action, v_F is a continuous signal which would be implemented using a PWM.

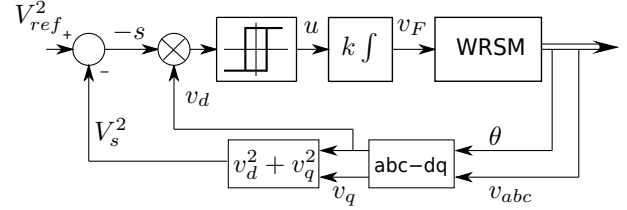


Fig. 3. Control scheme

IV. SIMULATIONS

In this section the simulations results using the designed controller are presented. The WFSM is a 40kW machine with the following parameters: $L_s = 26.25\text{mH}$, $R_s = 0.181\Omega$, $L_m = 25.29\text{mH}$, $L_F = 27.19\text{mH}$, $R_F = 0.1002\Omega$ and the mechanical speed is fixed at $\omega = 314\text{rad s}^{-1}$. Initial conditions holds $V_s = 400\text{V}$ and the resistive-inductive load is defined trough the values $R_L = 4\Omega$ and $L_L = 10\text{mH}$. As for the control action, $k = 1$, $u_1 = -10^5$ and $u_2 = 10^5$ and it is implemented by means of an hysteresis function.

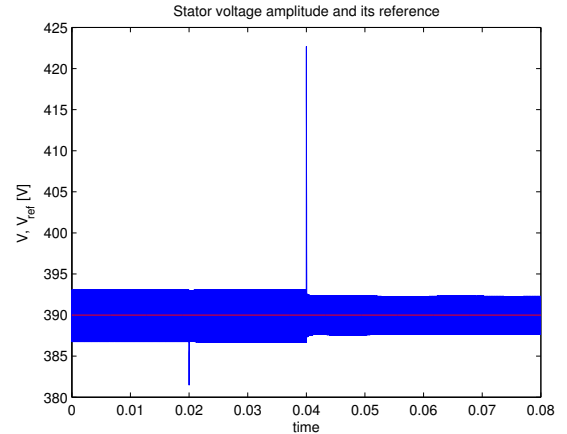


Fig. 4. Simulation results: stator voltage amplitude, V_s .

Two numerical experiments are run in order to evaluate the closed loop system performance: the load is perturbed in both, the resistive and inductive components, and some steps are considered in the reference. In the first test, load resistance R_L varies from 4Ω to 3.7Ω at $t = 20\text{ms}$ then, at $t = 40\text{ms}$, load inductance changes from $L_L = 10\text{mH}$ to $L_L = 5\text{mH}$.

The stator voltage amplitude, V_s , is depicted in Fig. 4. Note that, even R_L , L_L values are suddenly modified, the steady state value of the output is the desired one. Additionally, the convergence-time is small.

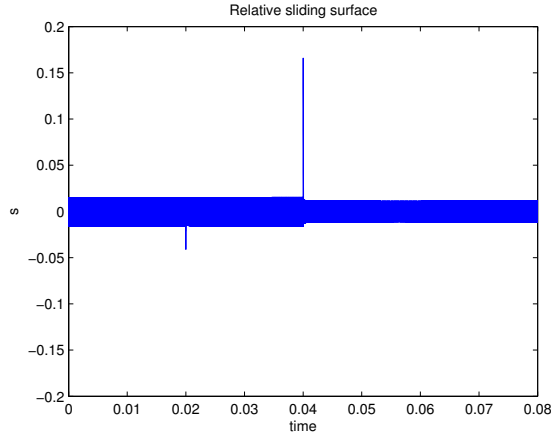


Fig. 5. Simulation results: sliding surface, s .

The sliding surface, s , is shown in Fig. 5. The selected values of u_1 and u_2 allow to keep the system on the sliding surface, which chatters around zero.

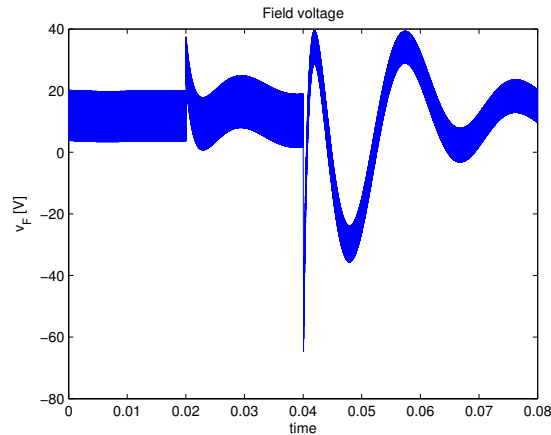


Fig. 6. Simulation results: field voltage and equivalent control actions, v_F , u_{eq} .

Fig. 6 shows the field voltage, v_F which remains in real implementable values. Note the dynamics of the control action is slower than the output dynamics. This is the effect of the ISD. Fig. 7 shows the three-phase stator voltages and currents, v_{abc} , i_{abc} and field current i_F , respectively.

In the second experiment the reference of the stator voltage amplitude is modified as Fig. 8 shows. The same figure shows that the desired stator voltage amplitude is reached in a short time.

Fig. 9 shows the sliding surface behavior which keeps near zero.

The field voltage, v_F remains in the expected boundary as Fig. 10 shows. The three-phase stator voltage and currents, v_{abc} , i_{abc} , and field current, i_F , respectively can be appreciated in Fig. 11.

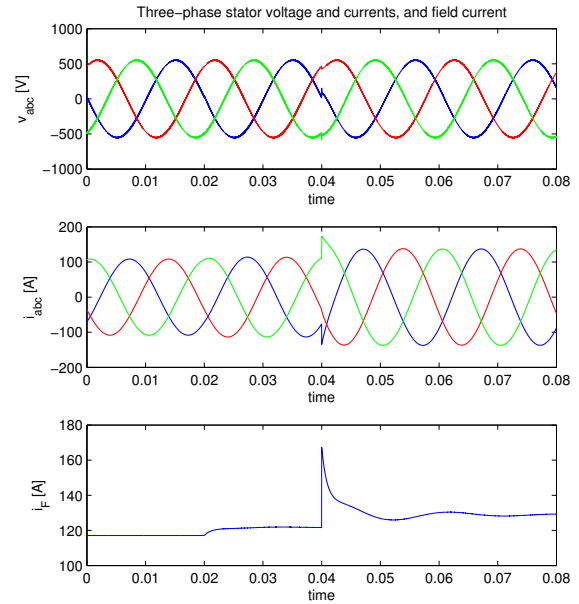


Fig. 7. Simulation results: three-phase stator voltage and currents and field current, v_{abc} , i_{abc} and i_F .

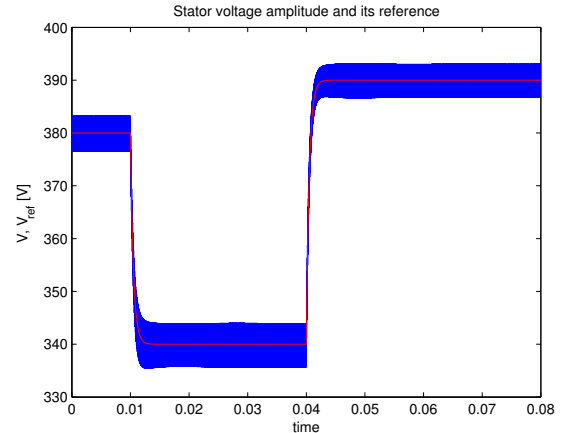


Fig. 8. Simulation results: stator voltage amplitude, V_s .

V. CONCLUSIONS

In this paper a Sliding Mode controller has been designed for a WRSM acting as a generator for an isolated resistive-inductive load. As result, a robust closed loop performance, which neither depends on the machine parameters nor on the load values, is obtained.

The stability of the closed loop system has also been numerically checked. Although it shows a rich nonlinear behaviour, the two equilibrium points are locally asymptotically stable.

Further research includes the study of nonlinear loads and the experimental validation of the controller in a real plant.

VI. ACKNOWLEDGMENTS

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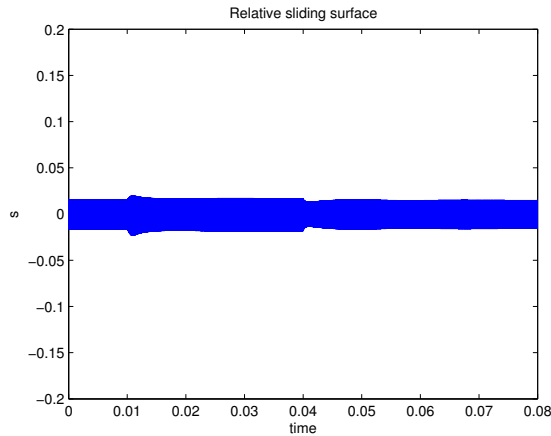


Fig. 9. Simulation results: sliding surface, s .

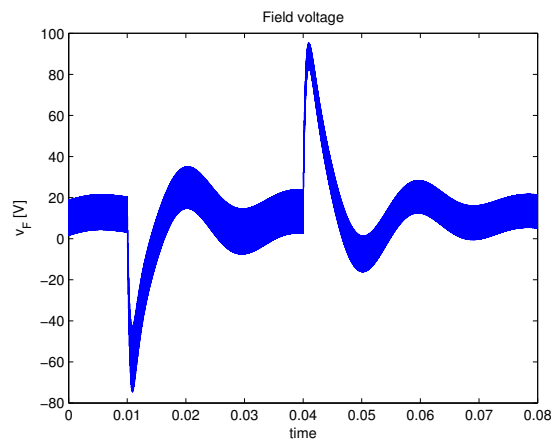


Fig. 10. Simulation results: field voltage control action, v_F .

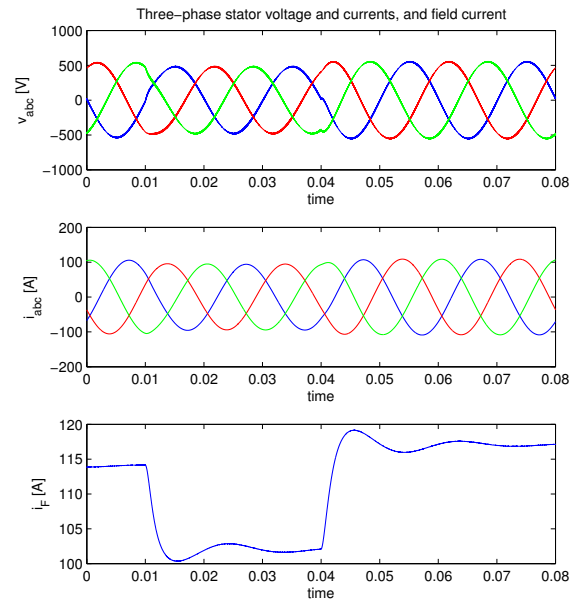


Fig. 11. Simulation results: three-phase stator voltage and currents and field current, v_{abc} , i_{abc} and i_F .

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